50X1-HUM

A New Method for Measuring the Thermal Constants of Wet Substances (2nd Report)

Japanese-language technical journal, TANASAWA Yasushi
Tokyo, Vol 1, No 3 (August) 1935

50X1-HUM

"A NEW METHOD FOR MEASURING THE THERMAL CONSTANTS OF WET SUBSTANCES" (REPORT #2)

TAKASAWA Yasushi,
TŌHOKU Imperial University.

(Note: The following report was delivered 25 November 1933 before a lecture meeting on applied dynamics, and later appeared in a Japanese-language technical journal dated August 1935. The table of contents is given first below.)

TABLE OF CONTENTS

AUTHOR'S ABSTRACT

This is a continuation of the first report, which was given in the same journal, Volume 35, Number 181 (May 1932). It discussed a method for measuring thermal conductivity by employing a ~~frequency~~ period of 60s and a temperature wave of maximum amplitude 0.2°C while at the same time preventing the movement of moisture. After that report, we improved the apparatus for measuring thermal conductivity and enhanced its accuracy, mainly by maintaining the correct frequency of interruption of the heating current and keeping fixed the average temperature. At the same time we investigated mathematically the movement of moisture and the transient-state terms which are due to various error-causing influences - namely, (1) the influence caused by the temperature waves's not being a pure sinusoidal wave; (2) the terminal effects of the two end faces of the cylinder; (3) the initial distribution of temperature. We consider methods for removing these causes.

We not only attempted by experiments to find to what extent the transient-state terms and the temperature wave's ~~frequency~~ period and amplitude change during subjection to heat and whether the moisture moves or not, but also sought the relation between (a) moisture content and porosity and (b) thermal conductivity in test samples of porous molding sand, by varying the heating current and frequency and holding constant the density during drying and the average temperature.

From the fact that all the experimental results lie well on the same curve we confirmed the absence of movement of moisture and learned that the thermal conductivity of molding sand increases abruptly if the moisture is initially absorbed, but becomes very great for a certain moisture content and then decreases, on the contrary, at lower values of moisture content. We found that the rate of heat transmission increases with moisture content rapidly at first and then slowly and finally becomes several times that of dry sand. The rate of heat transmission in dry sand was found by methods other than the ~~frequency~~ period method -- namely, by the comparison, cylinder, and injection methods. For the sake of verifying the accuracy of the experiments we compared the values due to these methods and found that there was an accuracy of 1.4%. The above results are in agreement with early practical knowledge and common sense.

INTRODUCTION

When an attempt is made to determine the rate of heat transmission by the usual method of measurement the moisture ~~content~~ gradually moves because of the steam pressure from the high-temperature portion to the low-temperature portion (because there must be such differences in temperature in the test material), thus making the measurements difficult.

When short-period low-amplitude temperature waves are transmitted through the test material and the phase difference of the waves at two different points are measured, each point within the test material is quickly and repeatedly heated and cooled and thus the average temperature can remain constant; therefore the author planned to seek the thermal conductivity under the condition where the movement of moisture has been stopped.

The first report described the nature of the temperature wave and the apparatus for measuring the thermal conductivity, but since then the apparatus and other things have been improved and the accuracy of measurement also has been enhanced; therefore a brief description of results will be given. For the sake of reference let us note the main points of the previous report: (a) We considered the various phenomena that express the time when the temperature wave passes through the material, especially in the case where the temperature wave comes from outside a cylinder. (b) From these results we deduced that in the cylinder-shaped test material purely sinusoidal temperature waves pass through with comparatively no damping; we then sought the phase difference by drawing the wave with the aid of resistance thermometers which are installed all around the cylinder and on the center line. (c) We then put an asbestos layer on the outside of the test material and analyzed the wave in the layer; the fundamental wave only passes through the layer, which serves to damp the high-frequency waves, thus permitting a purely sinusoidal temperature wave to be obtained.

1. THE APPARATUS FOR MEASURING THERMAL CONDUCTIVITY

In order to tamp the wet test material compactly (1) into the copper cylindrical tube shown in Figure 1, a piston (3) with small holes in the center line is used; when the test material has been made sufficiently hard by raising and lowering the piston, a resistance thermometer, which measures the temperature wave in the center (4), is carefully and accurately installed in the center portion.

By this method nonuniformity in the experiments is avoided, since maximum density of the powdery material is obtained and the density at the time of drying also is approximately fixed.

The temperature wave outside the test material is examined by means of a thermometer (5) attached closely to the outside of the copper cylindrical tube (2); the tube is wrapped with ~~an~~ analytical asbestos layer (6), and around this is wrapped heating nichrome wire (8).

The appearance of the new apparatus is shown in Figure 2. The points of improvement are summarized as follows:

(a) The test material is compactly packed to avoid nonuniformity of results.

(b) The alternating-current source has been eliminated, and a direct current from a battery is used to heat the nichrome wire (8), which is completely concealed in a steel tube to prevent electrical damage (7) (Figure 1).

(c) In order to maintain a constant average temperature, all the test material is inserted inside of a brass tube (9) and immersed in a large water tub (2) equipped with an agitator to keep the water at a constant temperature (Figures 1 and 2).

(d) In stead of an intermittent heating current from an electric motor, a black-painted semicircular disk attached to (6) the axis of a clock is used which interrupts periodically the beam of light to the photoelectric tube (5) connected to the amplifier (4) and the relay (3); and a constant current is passed through the nichrome heating wire periodically, thus producing the temperature wave. The ~~frequency~~ period can be varied with the range of 20 to 300 ~~cycles~~ seconds by adjusting the hair-spring balance wheel and the insert cog-wheel (Figure 2).

(e) In determining the time, the method where the lamp is lit and extinguished by contact between salt water and pendulum has been discontinued; the beam of light is interrupted by a clock pendulum with direct ~~frequency~~ period of 1s and is photographed on photographic printing paper (7) (Figure 2).

2. LIMITS OF DETERMINATION

It is desirable that the ~~frequency~~ period and amplitude be as small as possible, but the range is naturally limited by the experimental instruments that can be employed at the present time.

When recording the temperature variations by connecting the resistance thermometer (with resistance r_3 ohms) to the Wheatstone bridge as shown in Figure 3, the battery of voltage E volts should be so arranged that the relation $r_4 > r_3 > r_2 > r_1$ be fulfilled in order that good sensitivity may be obtained.

If the resistance of the galvanometer is r_g ohms and its sensitivity is i_g amp/mm and if i_g is proportional to $r_g^{\frac{1}{2}}$, then a 1-mm movement of a point of light on photographic printing paper which is at a distance of 1 meter from the galvanometer will be equivalent to

$$\delta T°C/mm = (i_g/a_T E)\cdot\tfrac{1}{2}n^{-\frac{1}{2}}(n+1)^2(r_1+r_2+r_3+r_4) \qquad (1)$$

(Note: this was derived from the formula for δT in the first report.)

Here n is:

$$n = r_g\cdot(r_1+r_2+r_3+r_4)/(r_1+r_3)(r_2+r_4) \quad . \qquad (2)$$

and becomes 1 when the r_g and the external resistance of connection are equal; the maximum value of $\frac{1}{2}n^{-\frac{1}{2}}(n+1)^2$ then becomes 2. a_T is the temperature coefficient of the resistance when T is given in °C; for pure copper it equals 0.00427 ohm/°C, and for pure silver it is about 0.0038 ohm/°C.

For example, if one uses YOKOKAWA Manufacturer's D_3D-type, $r_g \cong 10$ ohms and $i_g = 150/10^{10}$ amp/mm in the copper-wire thermometer with resistance $r_3 = 11$ ohms; and if one selects the values $r_1 = 1$ ohm, $r_2 = 5$ ohms, and $r_4 = 55$ ohms, then n becomes $n = 1$. E volts is limited by the heating effect of the current flowing in the resistance r_3 ohms, but in the above case it becomes at the most $E = 1$ volt; therefore if this arrangement is employed in the neighborhood of 0°C, we have from equation (1) the following:

$$\delta T = 150\cdot 10^{-10} \times 2 \times 72 / (4.27) \times 10^{-3} = 0.000503\ ^{\circ}C/mm . \qquad (3)$$

In the case of practical measurements R is made as small as possible; therefore under optimum conditions a 1-mm movement of the spot of light corresponds to $1\cdot 10^{-3}\ ^{\circ}C$.

Since the spot of light must show at least an amplitude of 10 mm, the temperature amplitude that can be measured in the temperature wave for a test sample becomes $1\cdot 10^{-2}\ ^{\circ}C$.

Table 1. The Minimum Value of F_o Corresponding to the Surface Temperature Amplitude T_m

T_m	0.1°C	0.2°C	0.4°C
Semi-Infinite Solid	0.60	0.35	0.26
Half Plate	0.34	0.22	0.17
Cylinder	0.19	0.13	0.10
Sphere	0.13	0.09	0.07

Table 1 results if one takes the above-mentioned temperature amplitude of $1\ 10^{-2}\ ^{\circ}C$ as the basis for deriving the minimum period that can be used when the test materials are: (A) semi-infinite solid, (B) half plane, (C) cylinder, (D) sphere.

Here $F_o = \kappa\tau / R^2$, kappa κ is the thermal conductivity, tau τ is the period, R is the distance from the surface to the point of measurement. If the surface temperature amplitude is given, F_o when minimum amplitude is 0.01°C is sought from the table; it is also understood that the period must be made long if the test material is a substance with small thermal conductivity.

In order to learn a practical value of the period tau τ, let us assume that we are measuring some insulator lagging such as carbonized cork whose thermal conductivity is $\kappa \cong 0.00072\ m^2/h = 0.002\ cm^2/s$.

In the measurements a small period can be used but errors of measurement increase if R is taken below 10 mm; therefore if R is 1 cm the relation between T_m °C and τ, the period in seconds, becomes as shown in Table 2:

Table 2. The Minimum Period τ That Can be Used in a Substance With $\kappa = 0.002\ cm^2/s$

Surface Temperature Amplitude T_m	0.1°C	0.2°C	0.4°C
Semi-Infinite Solid	300	175	130
Half Plane	170	110	85
Cylinder	95	65	50
Sphere	65	45	35

Finally, the period is strongly governed by the geometrical shape of the test object; the period must increase generally in the following ratios 2:3:5:8 for the following order/of shapes: sphere, cylinder, half-plane, semi-infinite solid. From this point of view the sphere is the best shape, and never has the so-called terminal effects to be discussed later; experimentally, however, it is difficult to limit the thermometer to the center point only, and in addition the lead wires intersect the isothermal surfaces, to which difficulty must be added the complexity of construction in heating. For these reasons we selected the cylinder instead of the sphere. Consequently, the minimum period in the case of measurements of insulator laggings is about 60 seconds if a maximum temperature amplitude of 0.2°C is permitted; it is very difficult to take this minimum amplitude below 30 seconds when the substances possess about average thermal constants.

3. ANALYSIS OF THE TEMPERATURE WAVE

We have already mentioned that when an arbitrarily shaped temperature wave is propagated inside a substance the high-frequency waves are noticeably damped so that only the fundamental wave is transmitted, which for all practical purposes finally becomes a sinusoidal wave; however we shall try to determine to what extent this is true.

Since the cooling, for small temperature differences, is proportional to this difference, the temperature wave that is produced when a constant current is interrupted periodically in a nichrome heating element assumes a negative exponential form of e. For example Figure 6 in the first report shows the curve of a weak electrical current with a period of 5 minutes, obtained in measurements made on a standard glass plate. This corresponds to the case where a is 0.5, which will be explained below. Figure 4(d) records by means of a thermocoupe the variations in temperature in the neighborhood of a heating element in the case of measurements with damp molding sand; because it was tamped in a steel tube, the induction effect due to current interruption in the neighborhood of largest and smallest values becomes evident in the form of a small wave.

The temperature wave near the heating element is called a surface wave. If the amplitude is taken as A, we have:

$$\left.\begin{array}{lll} \text{during heating} & y = 2A(1 - e^{-ax})/(1 - e^{-a\pi}) & 0 \leq x \leq \pi \\ \text{during cooling} & y = 2A(e^{-a(x-\pi)} - e^{-a\pi})/(1 - e^{-a\pi}) & \pi \leq x \leq 2\pi \end{array}\right\} (4)$$

Alpha a is a constant that depends on the electrical current and on the conditions of insulation, and becomes large if the current is weak and the period is made long.

Exp[illegible]

the value of a is close to 0.5 and in Figure 4(d) it is close to 0.

Analysing formula (7) and subtracting the constant A [in the first report]

we get:

$$y = \sum_{k=1}^{\infty} A \cdot \frac{4}{\pi} \cdot \frac{1 + e^{-a\pi}}{1 - e^{-a\pi}} \cdot \frac{a}{2k+1} \cdot \frac{1}{\sqrt{(a^2 + (2k+1)^2)}} \cdot \sin\left[(2k+1)x - \arctan\left(\frac{2k+1}{a}\right)\right] \quad (6)$$

If $a = 0.5$ then we have

$$y = 0.868A\left[\sin(x - 63.45^\circ) + 0.123\sin(3x - 80.55^\circ) + 0.0446\sin(5x - 84.30^\circ) + \ldots\right] \quad (5')$$

If $a = 0.0$ then we have

$$y = 0.812A \cdot (\sin x - 0.111 \sin 3x + 0.04 \sin 5x + \ldots) \quad (5'')$$

In these kinds of waves the second higher frequency does not appear, the ratio of A_3 the amplitude of the third harmonic and A_1 the amplitude of the fundamental wave is given as follows: $A_3/A_1 = (1/3) \cdot \sqrt{(1 + a^2)/(9 + a^2)}$. (6)

In order to simplify the initial explanation and the numerical calculations, let us consider the following case: an analytical layer, as in Figure 5, is placed on a semi-infinite solid material to be tested, with the constants $\lambda, c, \gamma, \kappa$ (the layer has a thickness of a and its constants are denoted by the subscript 1). It will be assumed that a purely sinusoidal wave is transmitted from the exterior of the analytical layer and that a stationary state has been attained.

If we assume the temperature at the point of contact ($x = a$) to be $\eta_a T_m \sin(wt - \phi_a)$, then we have

$$\left.\begin{aligned} \eta_a &= 2/\sqrt{c_1^2\cos^2(w/f_0)^{\frac{1}{2}} + c_2^2\sin^2(w/f_0)} \\ \phi_a &= \arctan\left[(c_2/c_1)\cdot\tan(w/f_0)^{\frac{1}{2}}\right]. \end{aligned}\right\} \quad (7)$$

Here $f_0 = \kappa_1 \cdot /a^2$

$c_1 = (1 + \sigma)\cdot\exp\,(w/f_0)^{\frac{1}{2}} + (1 - \sigma)\cdot\exp -(w/f_0)^{\frac{1}{2}}$

$c_2 = (1 + \sigma)\cdot\exp\,(w/f_0)^{\frac{1}{2}} - (1 - \sigma)\cdot\exp -(w/f_0)^{\frac{1}{2}}$

$\sigma = (\lambda c\gamma / \lambda_1 c_1 \gamma_1)^{\frac{1}{2}}$

$(\lambda c\gamma)^{\frac{1}{2}}$ stipulates the magnitude of heat transmission during the transient state, and is in contrast to λ the rate of heat transmission during the steady state. To put it crudely, if they are called tentatively the heat transport rate the quantities η_a and ϕ_a will differ depending upon the ratio of these rates and consequently the analyzability too will vary. Table 6 shows the results obtained for η_a and ϕ_a corresponding to various values of f_0 and σ ($\sigma = 0, 1, 2, 4$).

The effect of the analytical layer is shown by the ratio of amplitudes $(A_3/A_1)_{x=a}$ of the third harmonic wave and the fundamental wave at the point of contact; therefore if one calculates and graphs the influence of f_0 , σ at $a = 0$, that is, in the case of ~~trigonometric~~ saw-tooth surface waves, then one obtains the solid line in Figure 7. The greater the rate of heat transport, the more the analytical ability increases; but generally it becomes less than 1% of the amplitude of the fundamental wave if within $f_0 = 0.25$.

In the case of a cylinder as shown in Figure 8, the temperature at the boundary face is given by $\eta_m T_m \sin(wt - \phi_m)$ the imaginary part of the following:

$$[T]_{r=R} = T_m \cdot e^{iwt} \cdot b(R) \cdot \begin{vmatrix} b_1(R) & k_1(R) \\ b_1'(R) & k_1'(R) \end{vmatrix} \Bigg/ \begin{vmatrix} b_1(R_1) & k_1(R_1) & 0 \\ b_1(R) & k_1(R) & -b(R) \\ b_1'(R) & k_1'(R) & -\sigma b'(R) \end{vmatrix} \quad (8)$$

Here

$$k_1(R_1) = \ker\sqrt{\frac{w}{\kappa_1}}\,R_1 + i\cdot \mathrm{kei}\sqrt{\frac{w}{\kappa_1}}\,R_1$$

$$= \ker\sqrt{2\pi/f_0}\cdot(1 + R/a) + i\cdot \mathrm{kei}\sqrt{2\pi/f_0}\cdot(1 + R/a)$$

$$b(R) = \mathrm{ber}\sqrt{w/\kappa}\cdot R + i\cdot \mathrm{bei}\sqrt{w/\kappa}\cdot R$$

$$= \mathrm{ber}\sqrt{2\pi/f_0}\cdot\sqrt{\kappa_1/\kappa}\,(R/a) + i\cdot \mathrm{bei}\sqrt{2\pi/f_0}\cdot\sqrt{\kappa_1/\kappa}\,(R/a)$$

$$b_1'(R) = \left[\partial/\partial\left(\mathrm{ber}\sqrt{w/\kappa_1}\, r + i\cdot \mathrm{bei}\sqrt{w/\kappa_1}\cdot r\right)\right]_{r=R} \Big/ \sqrt{w/\kappa_1} \; .$$

Therefore we have relations between the ratio of the radii of curvature a/R and the ratio of thermal conductivity κ_1/κ, besides the rate of heat transport σ. The dotted line in Figure 7 shows the analytical ability for variables a/R and f_0 at $\sigma = 1$ and $\kappa_1/\kappa = 1$. The smaller a/R the more the analytical effect increases and the more the case of a semi-infinite solid is approach.

If asbestos paper is used as the analytical layer and the test material is wet sand, then we have $\sigma \doteq 4$; in the range $a/R = 0.5 \sim 1.0$ and below $f \doteq 0.25$ we have $A_3/A_1 = 0.01$.

The above concerned a relation between the surface wave and the temperature wave at the boundary face but the previous Figure 4 shows curves for experiments on wet sand to determine how the form of the surface wave varies when passing through the analytical layer and through the test material.

(d) is a temperature wave in the neighborhood of a nichrome wire; (c) is inside an anti-inductance steel tube; (a) is outside a copper tube; and, finally, (b) is the temperature wave that is recorded above the center line. However, the amplitudes are respectively according to these ratios since the two galvanometers regulated the voltage of the battery and thus variable sensitivity was used.

To determine the variation in this wave form see Figure 9. If relative to the fundamental wave y_1, the third harmonic wave y_3 ~~[illegible]~~ upon passing through the layer suffers a phase lag ... times as big ~~and the phase is retarded~~, then the ratio of amplitudes concave differ and gradually a sinusoidal wave is approached; actually, however, the phase lag is smaller than 3 times. Consequently the relative phase difference between y_3 and y_1 becomes small, and y_3 advances relative to y_1. As a result, the negative exponential curve changes its convex-upward portion to a concave-upward shape; a concave portion appears and thus a ~~[illegible]~~ saw-tooth wave is approached, with the front quickly becoming a pseudo-sinusoidal wave and the rear portion slowly assuming a pseudo-sinusoidal wave shape.

4 PHASE DIFFERENCE AND THERMAL CONDUCTIVITY

When a pure sinusoidal wave is sent through a cylindrical-shaped test material the phase lag ϕ_0 between the circumferential portion and the center becomes a function of $F_0 = \kappa\tau/R^2$ which lumps together κ, τ, R. Below let us consider how errors in the measurement of ϕ_0 is reflected in the thermal conductivity κ and also if F_0 may be determined from ϕ_0 when there is not a pure sinusoidal wave.

First, in order to investigate the effect on F_0 by ϕ_0, we derive the relation between dF_0/F_0 and $d\phi_0/\phi_0$, obtaining the following expression.

$$\frac{dF_0}{F_0} = \frac{2}{\sqrt{\frac{2\pi}{F_0}}} \cdot [illegible] \frac{bei'\sqrt{\frac{2\pi}{F_0}}}{ber'\sqrt{\frac{2\pi}{F_0}}} \cdot \frac{ber'^2\sqrt{\frac{2\pi}{F_0}} - bei'^2\sqrt{\frac{2\pi}{F_0}}}{[illegible]\sqrt{\frac{2\pi}{F_0}}\,ber'\sqrt{\frac{2\pi}{F_0}} - [illegible]\sqrt{\frac{2\pi}{F_0}}\,bei'\sqrt{\frac{2\pi}{F_0}}} \cdot \frac{d\phi_0}{\phi_0} \quad ([illegible])$$

$$= [illegible] \cdot \frac{d\phi_0}{\phi_0}$$

When $d\phi_0/\phi_0 = 0.01$, that is when ϕ_0 suffers 1% distortion, then F_0 according to Figure 10 varies in the range $0.5 > F_0 > 0.2$ and suffers a 1.7% distortion. If the measurements of R and tan τ are correct, then only kappa κ becomes affected.

Next, let us see what the results are if the wave is not a purely sinusoidal one. From the previous section, if the value $\kappa_1\tau/a^2 = f_0$ in the analytical layer is not taken below 0.25, then the third-harmonic wave around the cylinder remains about 1% at the least. Consequently, the temperature wave around the cylinder is a combination of the fundamental wave y_1 at $f_0 > 0.25$ and of the third-harmonic wave with a phase difference ϕ; there is no necessity of considering harmonics above the fifth. That is, the temperature wave is given by the following expression:

$$y = y_1 + y_3 = A_1 \sin x + A_3 \sin 3(x - \phi) \tag{10}$$

This combination wave y is intersected by the horizontal line $y = c$ as in Figure 11; if the angle included between the two peaks is taken as θ, then we have: $A_1 \sin x + A_3 \sin 3(x - \psi) = c = A_1 \sin(x + \theta) + A_3 \sin 3(x + \theta - \psi)$. Consequently, in order that $x + \frac{1}{2}\theta = \frac{1}{2}\pi$, we must have $\psi = 0$ ~~[illegible]~~ or $\theta = 120°$. Since psi ψ generally does not become 0, the angle theta θ, included between the two peaks ~~and~~ formed when the horizontal line intersects the wave, becomes equal to 180° and therefore the vertical line through half of θ passes through the summit of the fundamental wave. According to this method one can obtain without difficulty the maximum value of the pure sinusoidal wave which is composed of the 3rd harmonic wave. (Note: Generally we may take θ = 360/n when only one n-th harmonic is included.)

For the sake of reference, let us consider the method for obtaining the phase difference by taking the usual method when θ = 0; that is, taking the maximum value of the composite wave. Then, for the sake of brevity, take y in formula (10) as $y = A_1 \cos x' + A_3 \cos 3(x - \psi)$ (11)

and assume the range of values of x as $-\frac{1}{2}\pi \leq x \leq +\frac{1}{2}\pi$. The value x' of x when the maximum value of y is taken is obtained from the following expression:

$-A_1 \sin x' - 3A_3 \sin 3(x' - \psi) = 0$ (by differentiating the above and setting $dy/dx = 0$)

If x'' is taken as the value of x' when we maximize the difference between the maximum values of the fundamental waves, x = 0 and x', then we have

$$\sin x'' = \pm 3(A_3/A_1) . \tag{12}$$

Similarly we get

$$\sin x'' = \pm 1(A_3/A_1) \tag{13}$$

when we employ the method for obtaining the in-phase point at θ = 180°.

In these cases, Table 3 shows how far the phase of the composite wave is from the maximum value of the fundamental wave and when large, for various values of the amplitude ratio A_3/A_1.

Table 3 Maximum Error According to the Method of Determining Phase Differences

A_3/A_1	0.005	0.01	0.02	0.03	0.05	0.10
x''° (θ = 0°)	0.83	1.67	3.50	5.17	8.67	17.50
x''° (θ = 180°)	0.33	0.67	1.17	1.83	2.83	5.83

5. TERMINAL EFFECTS OF THE TWO FACES OF THE CYLINDER

Generally if an alternating current is used the terminal effect can be made small in comparison with the case for a steady current. (Note: see page 325 of the April 1932 issue of this journal.) Especially if F_0 is small it can be completely disregarded in the case of dried specimens, but in wet specimens if there are temperature differences at various points within the specimen due to the influence of the terminal effect the movement of the moisture will begin to occur;

therefore this must be guarded against no matter how small. Consequently the top and bottom end faces of the cylinder are tightly covered with lids, and the moisture content is prevented from escaping to the exterior; at the same time the average temperature at the various points in the specimen must be maintained always uniformly. We consider two methods: maintaining at a certain constant average temperature the end faces by employing some insulator material for the lids; and keeping the average temperature constant while letting the temperature of the end faces fluctuate by selecting some good conductor like copper as the lids.

Since the numerical calculations are simple let us see the effect of the top and bottom faces on the horizontal planes. When the lids are made of insulating material and the top and bottom faces are kept at a uniform temperature, the center temperature T_c under quasi-stationary conditions is $\eta_c T_m \cdot \sin(wt - \phi_c)$ where eta is:

$$\eta_c = \Big[\Big(\sum_{n=0}^{\infty} (4/\pi)(1/2n+1) \cdot \sin(\pi(2n+1)/2) \cdot \cosh \xi_n \cos \zeta_n / (\cosh^2 \xi_n \cos^2 \zeta_n + \sinh^2 \xi_n \sin^2 \zeta_n) \Big)^2$$

$$+ \Big(\sum_{n=0}^{\infty} (4/\pi)(1/2n+1) \sin(\pi(2n+1)/2) \cdot \sinh \xi_n \sin \zeta_n / (\cosh^2 \xi_n \cos^2 \zeta_n + \sinh^2 \xi_n \sin^2 \zeta_n) \Big)^2 \Big]^{1/2}$$

and the angle phi ϕ is the arctangent of the following ratio (namely, the square root of the last term divided by the first term in the above expression for eta η_c):

$$\left[\frac{\sum [illegible] \sin \frac{\pi}{2}(2n+1) \cdot \dfrac{\sinh \xi_n \sin \zeta_n}{\cosh^2 \xi_n \cos^2 \zeta_n + \sinh^2 \xi_n \sin^2 \zeta_n}}{\sum [illegible] \sin \frac{\pi}{2}(2n+1) \cdot \dfrac{\cosh \xi_n \cos \zeta_n}{\cosh^2 \xi_n \cos^2 \zeta_n + \sinh^2 \xi_n \sin^2 \zeta_n}} \right] \qquad \text{..................(14)}$$

Here xi ξ_n and zeta ζ_n are given by the following expressions:

$$\xi_n = \frac{1}{\sqrt{2}} \left(\sqrt{(n\pi R/L)^4 + (2\pi/F_o)^2} + (n\pi R/L)^2 \right)^{1/2}$$

$$\zeta_n = \frac{1}{\sqrt{2}} \left(\sqrt{(n\pi R/L)^4 + (2\pi/F_o)^2} - (n\pi R/L)^2 \right)^{1/2}$$

If we compute ϕ for several values of the ratio L/R and for $F_o = 1$, we obtain Table 4; in every case ϕ_c came out small. An error in the phase difference is revealed when compared in the case where L is very large.

Table 4. The Influence on the Phase Difference of ϕ_c Between the Top and Bottom End Faces

F_o	L/R	ϕ_c	$100 \cdot (\phi_c - \phi_o)/\phi_c$
1.00	2	84.633	-17.189
1.00	4	100.584	- 1.582
1.00	6	101.167	- 1.011

CONFIDENTIAL

If the lids are good conducters, then we have to superimpose on the above-mentioned solution the solution $\eta_c' T_m \sin(wt - \phi_c')$ in which $T_m \sin wt$ is taken as the temperature of the top and bottom faces and the temperature on the side surface is taken as the average temperature; therefore we obtain the following expression:

$$\text{coefficient of amplitude} = \left[(\eta_c \cos\phi_c + \eta_c' \cos\phi_c')^2 + (\eta_c \sin\phi_c + \eta_c' \sin\phi_c')^2 \right]^{1/2}$$

$$\text{phase difference} = \left[(\eta_c \sin\phi_c + \eta_c' \sin\phi_c') / (\eta_c \cos\phi_c + \eta_c' \cos\phi_c') \right] \quad .....(15)$$

η_c' and ϕ_c' have the same form as in equation (14), but without carrying out actual numerical computations we see that they become, on the contrary, almost the same value when for around $L/R = 6$ the value of F_o on the lateral face is 9 times that on the bottom face and for $\eta_c' \neq 0$ the value of ϕ_o is maintained at the average temperature.

In the case of a cylinder the terminal effect becomes still smaller in comparison with the above mentioned horizontal planes. Finally, measurements on samples that give an error less than 1% in ϕ_o the phase difference of the central portion have already been published for a semi infinite solid when $L \geq 6R + l$. (Note: see page 325 of the April 1932 issue of this journal.) Here the letter l stands for the effective length of the resistance thermometer. During actual experiments, rather than to insulate the top and bottom end faces and maintain them at the average temperature, it is easier to cover them with good conductors without changing the average temperature of the two end faces and to hold the average temperature inside the test material constant. therefore this method was chosen, and when we take $L/R \geq 10$ there is sufficient safety.

6. TRANSIENT PHENOMENA

During the transient period up to the time when the temperature distribution in the test material reaches the oscillatory quasi-stationary state, the movement of the moisture ~~content~~ occurs due to the temperature difference in the test material. But if the gap in the cylinder holding the test material is perfectly sealed tight with enamel or other substance, the moisture content is ~~stopped~~ in the test material; therefore as the quasi-stationary state is approached the average temperature becomes uniform and the moisture ~~content~~ again returns to a uniform distribution.

Since it was not desired during the experiments to have the moisture ~~content~~ ~~content~~ move even during the transient period, below we shall examine the influence of the various transient terms which disrupt the uniform temperature distribution.

CONFIDENTIAL

CONFIDENTIAL

If the main formula for the transmission of heat through a cylinder is solved for the boundary condition $(T)_{r=R} = T_m \cdot \sin wt$ and for the initial condition $(T)_{t=0}$, then besides $\eta T_m \cdot \sin(wt - \phi)$ we must add the following expression:

$$\eta' T_m = 4T_m \cdot \pi \cdot F_0 \cdot \sum_{n=1}^{\infty} \xi_n J_0(\xi_n r/R) \cdot e^{-\xi_n^2 \cdot m \cdot F_0} / (\xi_n^4 F_0^2 + (2\pi)^2) J_1(\xi_n) \quad \ldots\ldots (16)$$

Here ξ_n are the zeroes of $J_0(\xi) = 0$, and $m = t/\tau$, $F_0 = \kappa\tau/R^2$.

This term is the transient term which expresses the state that prevails until each point in the cylindrical test material, which is maintained at the mean temperature, attains the quasi-stationary state. Since this influence makes the temperature wave asymmetrical, it is accompanied by the movement of the moisture content if it is continued long. Since it is generally not attenuated at the center of the cylinder, its value there is found by setting $r = 0$.

Figure 4) shows the result for the case $F_0 = 1.0$ of graphing the following expression:

$$\eta_0' T_m = 4T_m \pi F_0 \sum_{n=1}^{\infty} \xi_n \cdot e^{-\xi_n^2 m F_0} / (\xi_n^4 F_0^2 + 4\pi^2) J_1(\xi_n) \quad \ldots\ldots\ldots\ldots\ldots (16')$$

If the temperature wave comes just once and no more, this term becomes very small; only if it is kept at the mean temperature from the very start of the experiment is the quasi-stationary state suddenly reached.

Next we shall consider the transient term at the time when the test material is immersed in the thermostatic tank (constant-temperature tank) or when a heating current just begins to flow. Since the temperature of the water tank must be very close to the mean temperature, a temperature difference is created in the direction of the radius when the test material is immersed in the tank. But in contrast to this, the asphalt analytical layer functions as a protective layer in this case also since its thermal constants are generally small in comparison with those of the test material, and although a temperature gradient exists in the analytical layer, it can be consider- very slight in the test material. Just how much can be found by computations, but it is preferable to regulate the intermittent current sent through the heating element, simultaneously with immersion in the tank, in such a way that while the difference in temperatures between the outer part of the test material and its center is being measured experimentally the temperature difference is made zero. The opinion in the first report that a strong initial current should flow in order to lead quickly to the quasi-stationary state at this point was in error.

When the transient term in Figure 3 has already become almost steady, the temperature of the water in the thermostat tank varies somewhat; but since its

CONFIDENTIAL

CONFIDENTIAL

period is long and amplitude is small there cannot be so much influence upon the temperature difference in the direction of the radius.

7. PERIOD AND AMPLITUDE OF THE TEMPERATURE WAVE, AND THE MOVEMENT OF MOISTURE

Because the velocity of diffusion of steam figures but slightly in the results of the experiments, it is difficult to discover relations between the amount of movement of the moisture [redacted] and the two quantities T_m (temperature amplitude) and τ (period); next, however, we shall attempt approximate calculations on the basis of several hypotheses.

If the surface temperature in a semi-infinite body varies as $(T_0 + T_m \sin wt)$, then the amount of steam that issues each half period $\tau/2$ at the surface is the following expression in units of kg/m^2:

$$\left.\begin{aligned} Q &= \sqrt{(2/\pi)}\sqrt{D\tau}\ \Delta\gamma \\ &= 0.8\sqrt{D\tau}\ \Delta\gamma \end{aligned}\right\} \qquad \ldots\ldots\ldots \quad (17)$$

Here we have assumed that the quasi-stationary state is the one involved, the steam closely follows the temperature, the saturation point is attained instantaneously, and the sum of the partial pressure of steam and the partial pressure of air (that is, the total pressure) is constant. The symbols used in this formula are defined thus:

$D\ m^2/h$ = rate of diffusion of steam relative to air; the total pressure in the solid is atmospheric pressure, and for mean temperature $T_0 = 40°C$ is taken as 0.12.

$\Delta\gamma\ kg/m^3$ = the density difference of saturate steam at T_m

Now since $T_0 = 40°C$, $T_m = 0.2°C$, τ = 1min = 1/60 hour, we have

$$Q = 0.8\sqrt{0.12/60}\ \ 0.0008$$

$$= .000029\ kg/m^2$$

$$= .0000029\ g/cm^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17')$$

This is multiplied in the case of the cylinder by the coefficient of shape (configuration) zeta ζ'. If $D\tau/R^2 < 1$, then ζ' is taken as $\zeta' = 0.9$ when between 1 and 0.9; therefore we have

$$Q = 0.0000026\ g/cm^2 \qquad \ldots\ldots\ldots\ldots \quad (17'')$$

Since Formula (17") merely indicates roughly the order of magnitude, it is actually better to determine it experimentally.

8. PREPARING THE EXPERIMENTAL DATA

As for the test materials, we selected porous casting sand in which the moisture [redacted] could be made easily to move. We attempted to find the relation between the the thermal constants and the moisture content (specific moisture).

CONFIDENTIAL

As for the size of the granules (granularity), the casting sand passed through a No 70 sieve but not through a NO 100. The casting sand chosen was washed repeatedly in water and small broken fragments adhering in star-shaped groups were eliminated.

According to the microscope, the size of the granules were all nearly uniform, with lumpy granules of average diameter of 0.17 mm being the most abundant; sometimes exceptional oval rod-shaped granules were seen under the microscope and having long diameter 0.34 mm and short diameter 0.09 mm. The composition of the sand was quartz with 1 part transparent to 4 parts nontransparent black and red in color. The test material was packed tightly by a piston into a round copper tube with the measurements shown in Figure 14.

9. DENSITY, POROSITY AND MOISTURE CONTENT OF MOIST SAND

The true density of the test material γ_s was found at ordinary temperatures to equal $\gamma_s = 2.515$ gm/cm^3 by measurement in a gravimetric bottle evacuated to a high degree by a vacuum pump.

Now if we use the following designations in the measurements:

- R cm = inner diameter of the round copper tube
- L cm = total length " " " " "
- m gm = weight of dry sand in " " "
- W gm = weight of water contained

we then obtain the following expressions, from which the composite states of water, air, and sand are clearly found:

volume inside the round copper tube $V\ \text{cm}^3 = \pi R^2 L$ . . . . (18)

density $\gamma\ \text{g/cm}^3 = m + W/V$ . . . . (19)

specific* moisture content (standard when dry) $\mu = W/m = \mu'/(1 - \mu')$ . . . . . (20)

specific moisture content (moist standard) $\mu' = W/(m + W) = \mu/(1 + \mu)$ . . (21)

volumetric moisture content $\nu = \mu\gamma_0$ . . . . (22)

density when dry $\gamma_0\ \text{g/cm}^3 = \gamma/(1 + \mu) = \gamma/ 1 + \frac{\mu'}{1 - \mu'}$ . . (23)

porosity $p = 1 - \gamma_0(\mu + 1/\gamma_s) = 1 - \gamma_0(1/\gamma_s + \frac{\mu'}{1 - \mu'})$ (24)

specific (gravimetric) saturation moisture content (dry standard) . . . . (25)

$$\mu_0 = 1/\gamma_0 - (1/\gamma_s) \cdot \mu = \mu'_0/(1 - \mu'_0)$$

ditto (wet standard) $\mu'_0 = (1/\gamma_0 - 1/\gamma_s)/(1 + 1/\gamma_0 - 1/\gamma_s) = \mu_0/(1 + \mu_0)$ (26)

Here porosity p indicates the volume of air after subtracting the volume of water $\mu\gamma_0$ and the volume of sand γ_0/γ_s contained in unit solid volume; the saturation moisture content expresses the condition where the moisture has driven out all the air in the sand. The measurement of m was by a large Sartorius balance which can measure from 10 000 grams to 0.0002 gram; W was found by measurement on the usual chemical balance, by taking about 3 grams directly after the experiment from the middle and end

*(Note: That is, relative to unit weight)

of the cylindrical-shaped test materials and drying at 105°C for 5 hours in a drying oven.

10. THE THERMAL CONDUCTIVITY OF MOIST SAND

If we take τ sec = the period of the temperature wave used, and

$\phi_o°$ = the phase lag in the temperature wave between the center and outside of the test material as determined on photographic printing paper

then we have the thermal conductivity given by $\kappa = F_o R^2/\tau = f(\phi_o°)R^2/\tau$ . . . . . (27)

We seek the mean temperature T°C at this time from the resistance R ohms of the copper resistance thermometer. If we compute beforehand the resistance R_1 ohms at the time when the temperature is T_1°C, then we get:

$$T°C = T_1 + (R/R_1 - 1)(1 + 0.00427T_1)/0.00427 \quad \ldots\ldots\ldots\ldots \quad (28)$$

The causes of errors were considered in sections 3-7, but these are more easily attacked experimentally than by computations. First, among the errors peculiar to the measuring devices are the following:

When determining whether or not the wave is a pure sinusoidal wave after passage through the analytical layer by analyzing it under a vernier microscope, we found a distortion of less than 1%.

No terminal effects were noticed, since the resistance wires were wound in the manner of two coils opposite in action.

The error in measuring the phase on the photographic printing paper was avoided by employment of the 120° partition method.

As was stated in the first report, the errors caused by the distortions from incorrect internal positions due to the distortions in the material of the resistance thermometers on the center line, thickness, and positions were small. As for the thickness and material (see Figure 8), the radius of the bar was taken from R to R_1 and the test material from 0 to R; if we seek the temperature on the center line we get the following expression: $\eta_o T_m \sin(wt - \phi_o)$, which is the imaginary part of the following:

$$[T]_{r=0} = T_m \cdot e^{iwt} \cdot \begin{vmatrix} b_1(R) & k_1(R) \\ b_1'(R) & k_1'(R) \end{vmatrix} \Big/ \begin{vmatrix} b_1(R_1) & k_1(R_1) & 0 \\ b_1(R) & k_1(R) & -b(R) \\ b_1'(R) & k_1'(R) & -\sigma b'(R) \end{vmatrix} \quad \ldots\ldots \quad (29)$$

If we set $R/R_1 = 0.1$

$F_o = \kappa\tau/R_1^2 = 6.2832$

$\sqrt{2\pi/F_o} = 1$

$\kappa/\kappa_1 = 1$

then we obtain ~~the~~ $\phi = (0.238125 + 0.011441\ \sigma)/(0.986064 - 0.001682\ \sigma)$ . . . (30)

For $\sigma = 1$, we have $\phi_0' = 14.233°$, which is in agreement with the results of the first report. Table 5 shows the variation of ϕ_0 with σ as computed from Formula (30); if a steel bar is inserted in the center portion, then sigma σ becomes 11.35 relative to dry sand and therefore the phase lag becomes large. If a glass rod is used, then σ becomes 0.755 relative to moist sand and the lag becomes about 1.2% smaller.

Of course in the above-mentioned computations the thickness of the inserted rod was taken 1/10 of the diameter of the test material and $\kappa = \kappa_1$, but in these experiments in which the thickness of the rod was less than 1/20 the error was 1 part in several tens of the above-mentioned table. Since metal rods cause still more uncertainty, we selected a glass rod easy to work. The temperature measured in the determinations was not at $r = 0$ but at the surface of contact; namely, at $r = R$. The rods were thin and similar

Table 5 The Error in Phase Due to the Ratio of Heat Transport Rates Between Test Material and Different Materials Inserted.

sigma σ	ϕ_0'	Error in %
0.0	14.503	+ 4.57
0.5	13.900	+ 3.34
1.0	14.233	+ 0.00
5.0	16.060	1[illegible]
10.0	20.000	+ 40.5

(5) We shall attempt to compute the error that occurred because the thermometer was placed on the outside instead of on the inside of the copper tube. The thickness of the round copper tube was 0.001 m and the thermal conductivity of copper is about 0.38 m^2/h; therefore disregarding the curvature every time that copper was used we found, for tau τ = 60 seconds, $F_0 = 0.38/60 \times (0.001)^2 = 6330$. Consequently, in the case where the test material used is a substance whose thermal conductivity is smaller than that of copper, a phase difference in the thickness of the tube is not created, and the temperature waves somewhat nonconcentric in the analytic layer can only be corrected as to concentricity. Figure 15 is a photograph taken for the two cases where the thermometer was attached on the outside and inside of the round tube and the test material used was moist sand. One cannot discern any phase difference between the two cases.

The various causes that produce water movements during the experiments are the following:

(1) Because the temperatures at the top and bottom end-faces of the tube are not the mean temperature, thermo-couples were buried in three places in the test material; namely, top, middle, and bottom, where the determinations were made. But under quasi-stationary conditions a temperature difference of less than 0.08°C barely appeared.

(2) Because the transient terms make the temperature wave asymmetrical, we shall find its influence, although small according to section 6, from experiments; it can be expected that this asymmetry will have some effect similar to that of (3) below.

(3) Because the uniform temperature distribution in the test material is disrupted at the time it is immersed in the large water tank or when the heating current begins to flow, differential thermo-couples are inserted in the center and the sides of the test material which is immersed in a water tank whose temperature is 3°C lower and then the temperature difference between the center and the outside is read, for the purpose of determining to just what extent the temperature distribution is disrupted. In the first half of Figure 16 we see that according to curve (a) the outside is much higher than the center after 6~7 minutes, the temperature then becoming equal to 0.65°C. The second half of curve (a) shows the effect of the heat added; it is the curve obtained when a regular current flows through intermittently after the first half. Similarly, after 6 ~ 7 minutes a temperature difference of 0.9°C was created in the reverse direction, and after that gradually decreased as the mean temperature rose. Curve (b) shows the case where in order to make the temperature difference small a weak current is caused to flow at the same time that the test material is immersed in the water tank, the temperature difference is gradually made greater by regulating the current. By this method the temperature difference can be limited to the neighborhood of 0.2°C.

(4) Since the period and the amplitude are too large, one must judge them from the results of numerous experiments.

There are no limits to the errors caused by lack of care~~lessness~~. Let us try to find the lag epsilon ε in the phase of the galvanometer, which causes considerable error.

If we use the following designations:

τ = period of a pure sinusoidal temperature wave
τ_g = period of the galvanometer
n = the ratio τ / τ_g

and if we consider the forced steady-state oscillations, we obtain the lag

$$= \arctan\left(\frac{2n}{m(n^2-1)}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

m depends upon the conditions of damping; m becomes infinite ∞ if there is no attenuation (decay damping), becomes 1 for limiting (critical) damping. The value of epsilon ε versus m, n is shown in Table 6, and the wave on the photographic printing paper lags just this much from the actual temperature wave.

The galvanometer used in the experiments is a D_3D type made by the Yokogawa Company; therefore, $\tau_g \doteqdot 5$ sec, and if τ = 60 sec then n = 12 (or if τ = 90 sec then n = 18).

Table 6. Phase Lag ε_0 in Galvanometer Temperature Wave

n	m=1, N=0.732	m=2, N=1.464	m=3, N=2.196
10	11.433	5.766	3.850
12	9.533	4.800	3.200
14	8.167	4.100	2.733
16	7.150	3.583	2.400
18	6.367	3.584	2.133
20	5.716	2.866	1.916

Since the sensitivity of the galvanometer is proportional to the square of the period in general, one with too short a period is not used; also, since it is damped generally by the critical damping, the phase difference becomes unusually large. Consequently, in this method two galvanometers with the same period must be put in the same damping condition and thus their relative phase differences are always maintained constant. Figure 17 shows the case where two galvanometers are connected in series; there is no phase difference between the two temperature waves on the figure (both galvanometers recorded the same temperature wave).

Finally, we may investigate whether the moisture moves or not when the temperature wave's period and amplitude and the heating current are given various values. Here since we may vary in any way either tau τ or R in the expression $F_0 = \kappa\tau/R^2$, we used the method where (1) either the period is made to vary between 55 and 95 seconds, (2) or one photograph is taken under steady state and then after (item 2 continued) interruption of this state another photograph is taken of the new steady state, when the results are compared.

Figures 18 and 19 show the results of such a method. Figure 18 shows two photographs for the same test material taken according to the method under (2) mentioned; here the temperature waves were in almost the same places. We clearly see that the results are identical and that the transient term of heating has no influence. Figure 19 shows the case where the period is varied.

Figure 20 shows the experiment where the thermal conductivity kappa κ is found as a function of the dry-standard moisture content mu μ (on the abscissa axis), for constant dry density gamma $\gamma_0 = 1100$ kg/m^3 and temperature $T_0 = 45°C$.

The various symbols ⊖, ⊖, ⊠ indicate where the determinations were made for various periods; since they lie well on the same curve, we can say that within the limits of these experiments (maximum amplitude 0.2°C and periods 60 - 90 seconds) the movement of the moisture has been prevented.

When we examine the variation of kappa κ due to mu μ, we see that from the origin $\mu = 0$ to $\mu = 9\%$ (the moisture increases) the value of kappa κ also increases, but very sharply; however when μ passes 9 % the curve descends.

That is, in the neighborhood of $\mu = 9\%$ kappa κ is a maximum.

When $\mu > \mu_0 = 51.1\%$, the air bubbles (blow holes, vapor bubbles) in the casting sand are replaced entirely by water, and by now the three phases of solid-liquid-gas become two phases: solid and liquid and therefore we are outside the limits of our methods of determination.

11. THE RATE OF HEAT TRANSMISSION IN MOIST SAND

The rate of heat transmission in moist sand is found from the following formula:

$$\lambda = \kappa \cdot c \cdot \gamma \quad \text{(where lambda is units of } g/cm \cdot sec) \; . \; . \; . \; (32)$$

Here the specific heat of moist sand c (in units of cals/g °C) is given by

$$c = c_0 + \mu/(1+\mu) = c_0(1-\mu') + \mu' = c_0\gamma_0 + \nu/\gamma_0 + \nu \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (33)$$

The specific heat for the dry case c_0 is found by the ~~composite method~~ method of mixtures (according to unpublished report).

Figure 21 shows the variation of λ, κ, c with the volumetric (specific, i.e. relative to unit volume) moisture content nu ν (on the horizontal axis); although kappa has a maximum at $\nu \doteq 9\%$, lambda does not. Table 7 shows the measurements for lambda λ of dry sand by the comparison method, tube method , and insertion method.

Table 7. Rate of Heating of Sand Dry for 40°C

Method	Moisture Content $\mu\%$	Heat Transmission Rate λ
Period	0.[illegible]	0.159
Insertion	0.[illegible]	0.160
Tube	0.170	0.151
Comparison	0.[illegible]	0.149

Here, what we mean by the comparison method is the method where the heat transmission rate is compared with known standard tables; the tube method is the method where a heating wire is placed in the center and the loss of heat from this wire while at steady state conditions is measured; the insertion method is the method where a solid at uniform temperature is immersed in a thermostatic tank at some other temperature and kappa κ is sought by finding the temperature variation at a certain point during the transient state.

Let us consider the reasons for the variation of lambda λ the heat transmission rate with the moisture content as shown in Figure 21. Under the microscope it can be seen that when water is put in sand the surface tension causes a water film to be stretched over the contact faces between the various sand granules.

When this water film (aqueous membrane) stretches, the contact area increases and the contact heat resistance decreases thus causing lambda λ to become larger; however when the water becomes too abundant above a certain level the increase in the contact area relative to the increase in volume of the water is not so evident as in the beginning. Also, water is absorbed even in places other than the contact faces, and lambda λ does not increase at the initial rate. Consequently this experimental result that lambda λ increases initially rapidly and then later rises more slowly can be thought of as natural and commonsense. Still, in order to make this idea more concrete, consider the granules as spherical and packed together in regular order and then calculate the isothermal surfaces; in this way the above-mentioned experimental results can thus be theoretically derived (according to a report still unpublished).

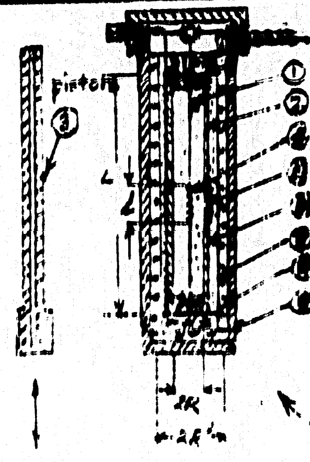
Since there are no other suitable methods we cannot learn the accuracy of these experimental methods, but it can be thought that we have an accuracy within ± 4% from the following facts: before and after the experiments there were no variations or differences in the moisture content; the same results were obtained even though the periods were varied and the conditions governing heating were made various; and the values for dry sand were close, showing a scatter of the experimental points of only ± 1% ~~in comparing~~ when the results of the various methods (comparison, tube, insertion, period were compared.

Finally, the author wishes to take this opportunity to thank Professor NUKIYAMA Shiro of the TŌHOKU Imperial University for his kind guidance. Part of the expenses for carrying out these experiments were defrayed by the financial assistance of the SAITŌ HŌON KAI (SAITŌ Gratitude Association) and the Imperial Academy.

E N D

(APPENDIX)

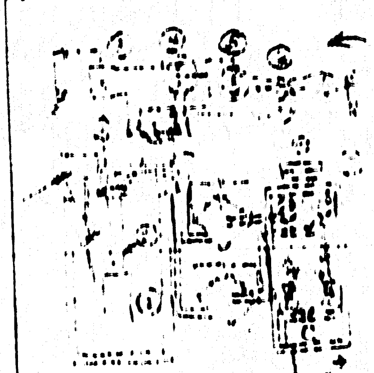
FIGURES



① Test Material
② Round Tube Made of Copper
③ Thermometer
④ Thermometer
⑤ Analytical Layer
⑥ Round Steel Tube
⑦ Heating Wire
⑧ Round Brass Tube

Figure 1. Receptacle for the Test Material

Figure 2. Entire Apparatus for Determining the Thermal Conductivity.



1. Test Material
2. Thermostat Tank
3. Current Interrupter
4. Amplifier
5. Photoelectric Tube
6. Light-Ray Breaker
7. Clock Pendulum

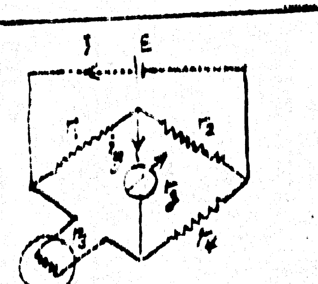


Figure 3. Wheatstone Bridge

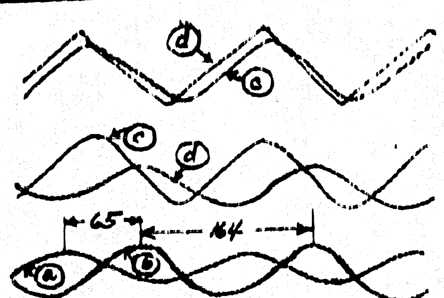


Figure 4. Modification in the Temperature Wave Due to Penetration into the Layer

a: above center line
b: above round copper tube
c: above round steel tube
d: near the nichrome heating wire.

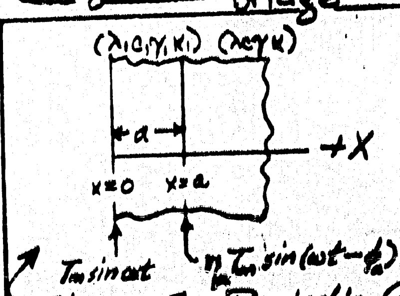


Figure 5. Periodic Conduction of Heat in the Two Layers.

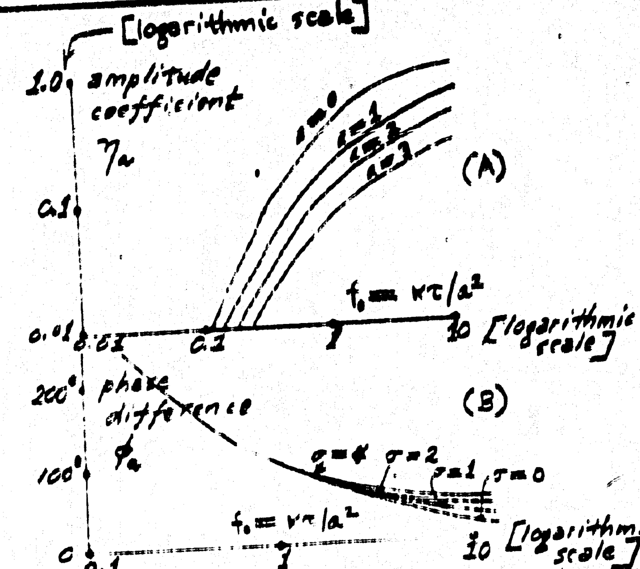


Figure 7. (A) Heat Transport Rate Ratio and Amplitude Damping Factor.
(B) Heat Transport Rate Ratio and Phase Difference.

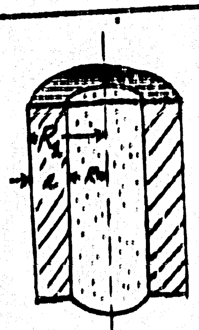


Figure 8. Two-Layer Round Tube

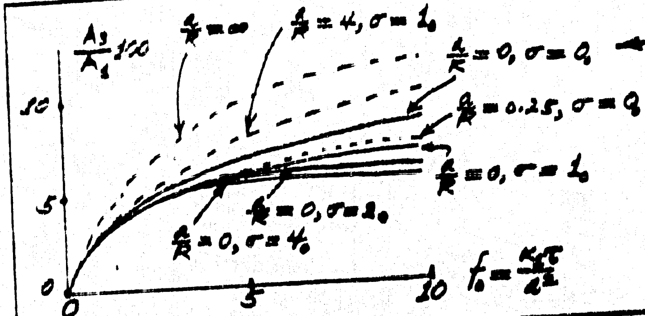
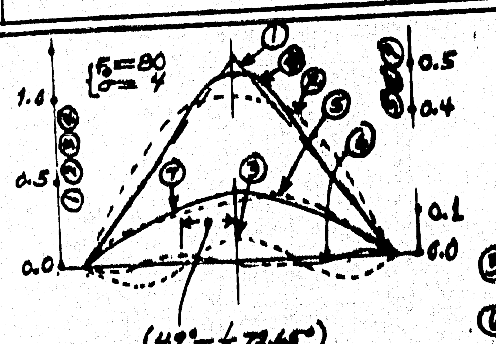


Figure 7. The Effect of Analysis (Resolution)



①: Saw-Tooth (Triangular) Wave

②: $0.812 \sin x = y_1$

③: $-0.812 \times 0.111 \sin 3x = y_3$

④: $0.812 \sin x - 0.812 \times 0.111 \sin 3x$

⑤: $0.193 \times 0.812 \sin(x - 49°) = y_1'$

⑥: $0.11 - 0.812 \times 0.111 \sin(3x - 72.65°) = y_3'$

⑦: $0.193 \times 0.812 \sin(x - 49°) + 0.11 \times 0.812 \times 0.111 \sin(3x - 72.65°)$

Figure 9 Analysis (Separation) of the Saw-Tooth (Triangular) Wave by the Analytical Layer

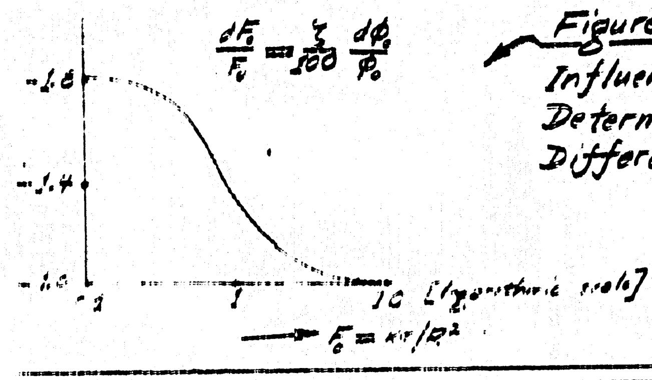


Figure 10. Influence of the Error in Determinations of Phase Differences upon F_0.

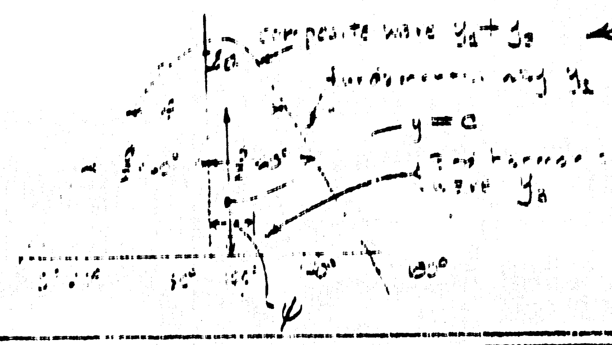


Figure 11 Finding the Top of the Fundamental Wave by Partition.

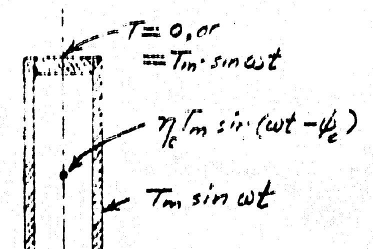
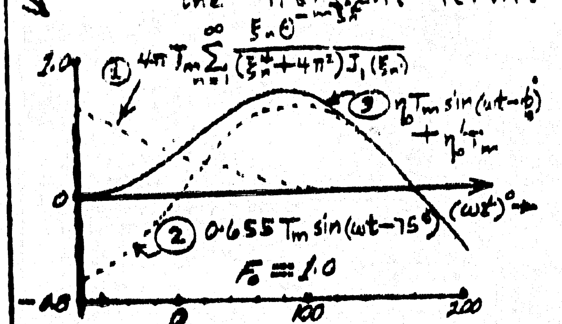


Figure 12

Figure 13. Distortion in the Temperature Wave Due to the Transient Term.



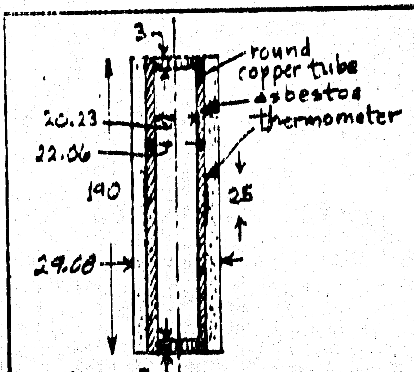


Figure 14. Measurements of the Round Copper Tube Containing the Test Material

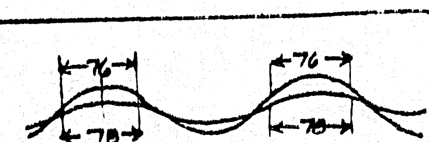


Figure 15. Phase Difference Between the Inside and Outside of the Round Copper Tube.

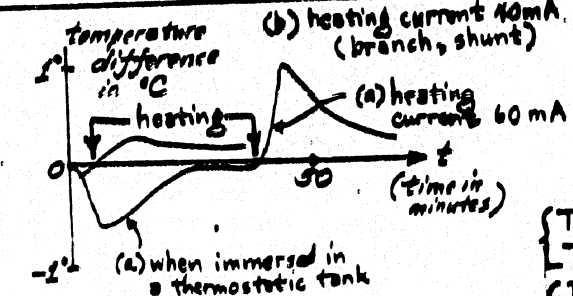


Figure 16. Temperature Difference During the Transient Period in the Test Material.

{Temperature of Thermostat Tank = 19.74 °C

{Initial Temperature in the Test Material = 22.30 °C

Figure 17. Lag Between the Two Galvanometers.

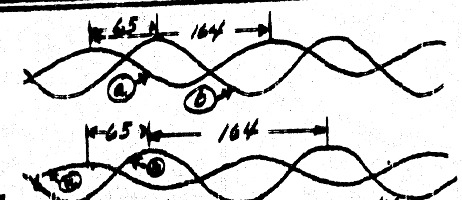


Figure 18. The Temperature Wave of ~~Average~~ Period Length.

(1) Influence of Transient State Due to Heating (τ = 82 sec, ν = 22.51 %, κ = 0.001, 68 m²/h)

(2) The result, after (1), when the quasi-stationary state is disrupted, the heating current is varied, and the quasi-stationary state is again resumed.

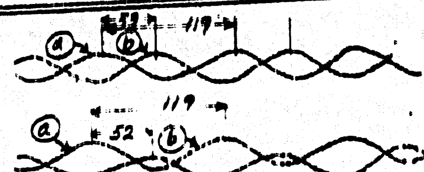
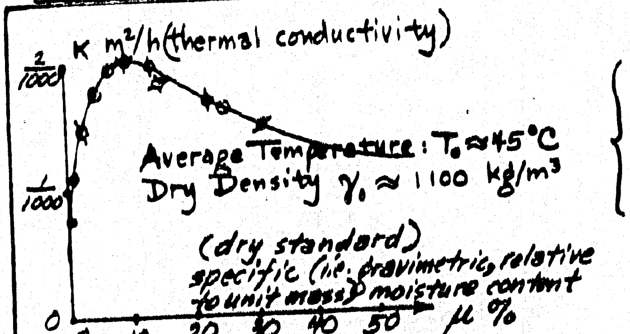


Figure 19. The Temperature Wave of Short Period

(4) τ = 59.5 sec, ν = 12.47 %
κ = 0.001, 89 m²/h.

(5) τ = 59.5 sec, ν = 9.07 %
κ = 0.001, 96 m²/h.



- : period $\tau \approx$ 94 sec
- : period $\tau \approx$ 82 sec
- : period $\tau \approx$ 60 sec

Figure 20. The Relation Between K (~~temperature~~ conductivity) and μ % (dry-standard specific* moisture content [*Note: that is, relative to unit weight.]

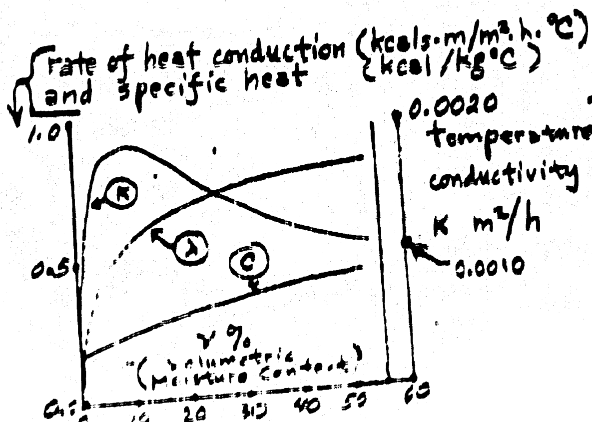


Figure 21. The Relation Between Thermal Constants and the Volumetric Moisture Content.

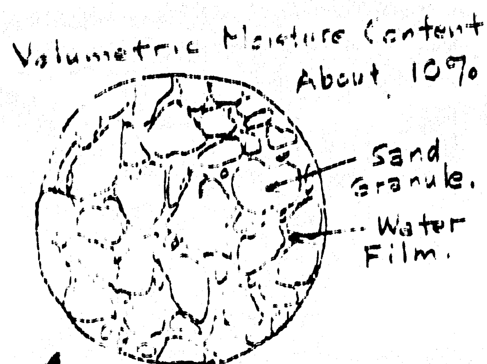


Figure 22.

Figure 22

E

END